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Groups of galaxies in the Local Supercluster: some hypotheses on the evolutionary stage

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Summary. We present a method capable of determining the evolutionary stage of bound, isolated systems of galaxies. In particular, we apply it to a sample of groups, belonging to the Local Supercluster, taken from the Geller and Huchra's (1983) catalogue. We find that most of them are still in the phase of collapse and not yet virialized. Their evolutionary stages are distributed around that of the Virgo cluster, which lies in a central position in the region considered.

Moreover, the knowledge of the evolutionary stage of a system permits the determination of its mass even when it is not in virial equilibrium. The median M/L ratio of the groups in our sample proves to be in the range (550–700) $hM_\odot/L_\odot(\mathrm{B})$ ($H_0 = h\,100\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$).

Key words: cluster of galaxies – cosmology

1. Introduction

We present a method capable of determining the evolutionary stage of bound, isolated systems of galaxies. In particular, we shall apply it to a sample of groups, belonging to the Local Supercluster, taken from the Geller and Huchra's (1983) catalogue. The knowledge of the evolutionary stage of a system of galaxies is interesting in the framework of theories concerned with the formation of large-scale structures in the universe. Moreover, if one knows the evolutionary stage, it is possible, at least in principle, to determine the mass of the system even when it is not in virial equilibrium, as well as to infer some properties of its initial condition.

In accordance with our method, we find that most of the groups in our sample are still in the phase of collapse and, thus, are not yet virialized. Their evolutionary stages are distributed around that of the Virgo cluster which lies in a central position in the region considered. The median M/L ratio of the groups in our sample does not change significantly with respect to the usual "virial" determination, even if it proves to be slightly larger, $M/L > 500\,hM_\odot/L_\odot(\mathrm{B})$ ($H_0 = h\,100\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$).

Section 2 introduces the method, while Sect. 3 describes the group sample we started with. Since some groups have crossing times comparable to the Hubble time, in Sect. 4 we discuss some criteria for determining whether they are bound or unbound systems. A short discussion of the uncertainties associated with the method is given in Sect. 5; in Sect. 6 we present the results given by our method, while conclusions are drawn in Sect. 7.

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2. The method

The kinetic energy T and the gravitational potential energy W of a system of total mass M are, as usual,

$$T = \tfrac{1}{2}MV^2, \tag{1}$$

and

$$W = -G\frac{M^2}{R}. \tag{2}$$

In the present paper, the groups are supposed to be composed mainly of galaxies (or of galaxies with cold dark matter halos); the internal degrees of freedom are neglected and assumed not to change during the evolution of the system. The velocity dispersion V and the virial radius R are obtained from the observed positions, radial velocities and luminosities of the component galaxies, considering the influence of observational errors as in Mezzetti et al. (1985), with a constant, morphology-independent, mass-to-light ratio and a minimum galaxy luminosity of $0.01L^*$.

The virial crossing time of the system is

$$t_{\mathrm{cr}} = \left(\frac{3}{5}\right)^{3/2} \cdot \frac{R}{V}. \tag{3}$$

Equations (1) and (2) give the mass of the system

$$M = \frac{1}{2\alpha}\frac{V^2R}{G}, \tag{4}$$

where $\alpha = T/|W|$. α may be considered a correction factor to the virial mass

$$M_{\mathrm{vir}} = \frac{V^2R}{G},$$

due to the incorrect application of the virial theorem to the system if the latter is not relaxed (in that case $\alpha = \frac{1}{2}$). For a gravitationally bound system, the total energy $E = T - |W| = |W|(\alpha - 1) < 0$, hence $0 \le \alpha < 1$ and $M > \frac{1}{2}M_{\mathrm{vir}}$.

The method presented in this paper permits the determination of α, once a model for the evolution of the system is assumed. It is not restricted to a particular choice of model and can easily be generalized to more sophisticated ones.

In the gravitational instability picture (e.g. Gunn and Gott, 1972), the initial density fluctuation grows, though lagging behind the cosmic expansion when it breaks away from the Hubble flow and begins to collapse. Later, a more or less violent (Lynden-Bell, 1967; Shu, 1978) relaxation process sets in, which prevents the system from collapsing into a point and instead brings it to equilibrium. The evolution of the typical system size is sketched in Fig. 2 of Gott and Rees (1975). The time evolution curves $\alpha(t)$, given by the dynamical models, are the starting point for the determination of the evolutionary stage.

Defining α_m and R_m as the values of α and R, respectively, at the maximum expansion of the system, the energy conservation principle allows us to write

$$V^2 = \frac{2GM}{R}\left[1 - (1 - \alpha_m) \cdot (R/R_m)\right], \tag{5}$$

while α can be expressed, using (4) and (5), as

$$\alpha = 1 - (1 - \alpha_m) \cdot (R/R_m). \tag{6}$$

When a system is virialized, its virial radius R_f is given by

$$R_f = \frac{R_m}{2(1 - \alpha_m)}, \tag{7}$$

so it is possible to express the virial crossing time (Eq. (4)), by means of (5), (6) and (7),

$$t_{cr} = t_{cr}(\mathrm{vir}) \cdot \frac{2(1 - \alpha)^{3/2}}{\alpha^{1/2}} \tag{8}$$

where $t_{cr}(\mathrm{vir})$ is the crossing time the system will have when virialized.

A convenient timescale for the system is its collapse time, defined as (see, e.g., Cavallere et al., 1978, 1986)

$$t_c = 2\pi\left(\frac{3}{10}\right)^{3/2} \cdot \frac{GM^{5/2}}{|E|^{3/2}}, \tag{9}$$

which, when the system is in virial equilibrium, equals $2\pi t_{cr}(\mathrm{vir})$.

It is known, from numerical simulations (see, e.g., Peebles (1970)), that a system reaches virialization in a time t_v approximately equal to $(3/2) \cdot t_c = 3\pi \cdot t_{cr}(\mathrm{vir})$. Thus, for a given evolutionary model of the system, α is a function of the ratio between the time Δt elapsed since the beginning of the expansion and t_v or $t_{cr}(\mathrm{vir})$.

The evolution curve of α used in this paper has been derived from numerical simulations made by Giuricin et al. (1984) for systems composed of 15 point masses with a Schechter-type mass function and $\alpha_m = 0$. In Fig. 1 we plot a smoothed version of the $\alpha(\tau)$ curve, where

$$\tau = \frac{\Delta t}{t_{cr}(\mathrm{vir})} = \frac{t_f - t}{t_{cr}(\mathrm{vir})} \tag{10}$$

is the time from the moment t_f of origin of the growing fluctuation, in units of $t_{cr}(\mathrm{vir})$. (Here and in the following, t denotes time counted backward from now, $t = 0$).

A bound system, whose size is proportional to $(1 - \alpha)$ (see Eq. (6)), typically expands approximately with the Hubble flow for $\tau < \pi < 2\pi$, then re-expands for $2\pi < \tau < 3\pi$, and eventually becomes relaxed for $\tau > 3\pi$ (see Fig. 1).

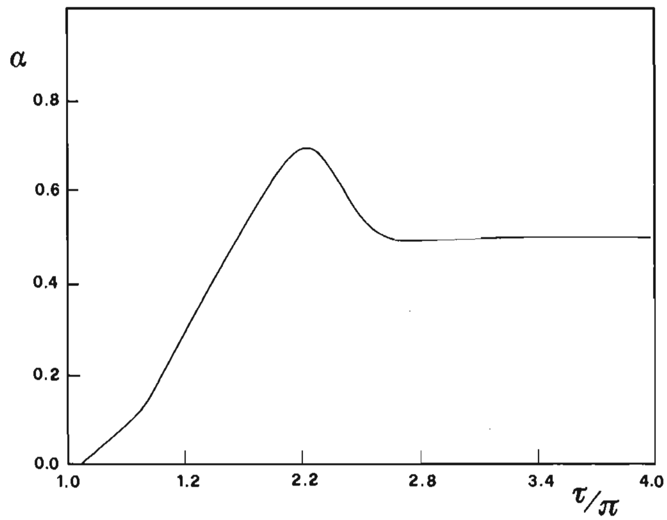

Fig. 1. The smoothed evolution curve $\alpha(\tau)$ used in the present paper, from Giuricin et al. (1984)

The gist of our method is to obtain the present ($t = 0$) value of τ (which we call the evolutionary stage) from the observed crossing time t_{cr} of the system (Eq. 3), by inverting the time evolution of t_{cr} given by Eq. (8). As there are models having $\alpha_m = 0$ and hence $t_{cr} = \infty$ (as in our case), it is numerically more convenient to use

$$\beta(\tau) = \frac{t_f - t}{t_{cr}} = \frac{\tau}{2} \frac{\alpha(\tau)^{1/2}}{[1 - \alpha(\tau)]^{3/2}}, \tag{11}$$

where the right hand side depends only on the model and the fluctuation formation time appears explicitly.

Figure 2 represents the $\beta(\tau)$ curves for the model we have adopted. For values of β having a 3-fold solution to Eq. (11), all three possibilities have been analyzed (see Sect. 6).

To invert Eq. (11) for the evolutionary stage, a value for t_f must be chosen to compute $\beta(\mathrm{now}) = t_f/t_{cr}$. In a standard Friedman model t_f changes less than one part in a thousand when its corresponding redshift z_f decreases from infinity to ≈ 100 (Table 1). This is tantamount to saying that for all those perturbations that begin to grow at $z_f \geq 100$ (a plausible range in the gravitational instability picture) a negligible error is made by assuming $t_f = t_0$, the present age of the universe.

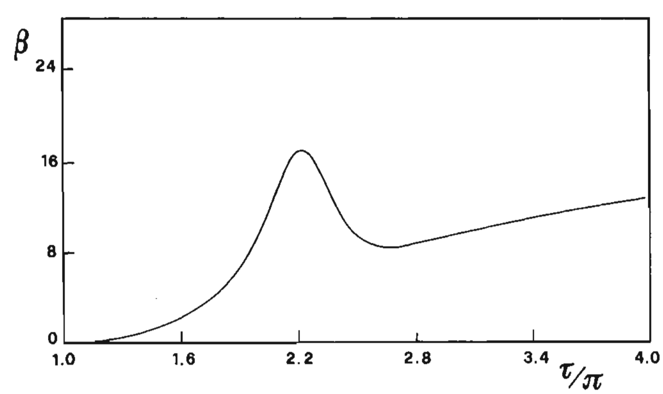

Fig. 2. The time evolution $\beta(\tau)$ for our model

Table 1. The redshift z_f corresponding to 1% and 0.1% deviations of the formation time t_f from the age of the universe t_0

$\Omega = 0.1$			$\Omega = 1.0$		
$\frac{t_f - t_0}{t_0}$	$H_0 t_f$	z_f	$\frac{t_f - t_0}{t_0}$	$H_0 t_f$	z_f
0.010	0.889	30	0.010	0.660	15
0.001	0.897	160	0.001	0.666	75
0.000	0.898	∞	0.000	0.667	∞

3. The sample

We have applied the method described in the previous section to groups of galaxies in Geller and Huchra's (1983) catalogue with distance D_v from the Virgo cluster (determined as in Giuricin et al., 1986) in the range

$$5.5\,h^{-1}\,\mathrm{Mpc} < D_v < 20\,h^{-1}\,\mathrm{Mpc},$$

where the upper limit defines the membership in the Local Supercluster and the lower limit is chosen to avoid the possible gravitational influence of the Virgo cluster, which is the most massive system in that region.

We discard group 56, which according to Giuricin et al. (1986) contains many interlopers; groups 57 and 79, which have large observational errors in the redshifts (Mezzetti et al., 1985); and group 22, for which the correction for the virgocentric infall may be questionable. (In the present paper we use Geller and Huchra's (1983) notation.)

To infer the dynamical status of a system of galaxies, positions and redshifts of a large fraction of the system components ought to be observed; otherwise, the determination of the relevant dynamical parameters would be affected by large uncertainties. As an indicator of our knowledge of the system, we have adopted the ratio, L_v/L_t, between the sum L_v of the luminosities of the observed galaxies and the total luminosity L_t of the system, estimated with the aid of the Schechter (1976) luminosity function as given by Geller and Huchra (1983). All the groups with $L_v/L_t < 0.75$ (groups 45, 62, 84, 86, 140 and 147) have been excluded from our sample.

The dynamical models we have considered in the application of our method presuppose that the evolution of the system is determined by internal gravitational forces and is not affected by tidal interactions with other neighbouring systems. As an indicator of the ratio of internal to tidal gravitational forces we have taken

$$\Gamma = \left[\frac{D_{ij}}{R_i + R_j}\right]^2,$$

with D_{ij} being the distance between the groups and R_i being a measure of the spatial extension of the single group (the mean pairwise separation). We have therefore excluded from our sample those groups which have $\Gamma < 1$ (groups 50, 51, 75 and 115), the others having $\Gamma \cong 10^2$.

30 groups from Geller and Huchra's (1983) catalogue constitute our preliminary sample; they are listed in Table 2, which also contains the values of t_{cr} in units of H_0^{-1}. We notice immediately that if all the groups in our sample were in virial equilibrium and thus lay on the right straight part in Fig. 2, they should have $t_{cr}H_0 < 0.1$, corresponding to the second minimum

Table 2. The groups of galaxies in our sample, selected from Geller and Huchra's (1983) catalogue. n_g is the group number, N is the number of observed members, D_v and D are the distances from the Virgo cluster and from us, respectively, in h^{-1} Mpc ($H_0 = h\,100\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$), t_{cr} and t_J are the observed crossing time and the Jackson crossing time, respectively, in units of H_0^{-1}, $\langle\theta\rangle$ the angle defined in Sect. 4; Jackknife estimates of standard deviations, σ_{JK}, are also given

n_g	N	D_v	D	$H_0 t_{cr}$	σ_{JK}	$H_0 t_J$	σ_{JK}	$\langle\theta\rangle$	σ_{JK}
43	5	15.7	20.1	0.47	0.32	0.25	0.10	61.0	10.9
44	3	12.7	9.3	1.81	–	0.19	–	70.7	10.2
46	4	18.0	19.0	0.23	0.17	0.44	0.53	67.2	23.8
52	5	13.8	15.9	0.09	0.49	0.15	0.26	74.7	14.7
58	10	8.4	14.3	0.05	0.04	0.15	0.06	66.4	8.4
60	4	9.0	15.9	1.41	0.20	0.52	0.15	49.0	11.1
61	3	8.0	15.5	0.43	–	0.31	–	68.4	9.6
63	4	16.5	18.6	0.24	0.17	0.10	0.05	84.2	5.6
64	3	15.9	14.9	1.85	–	1.32	–	55.0	18.0
66	3	19.2	31.7	0.84	1.77	0.25	0.31	65.6	20.1
67	8	10.4	19.3	0.05	0.09	0.18	0.06	71.2	7.0
68	23	6.4	11.9	0.10	0.08	0.21	0.03	66.6	3.2
71	7	7.9	16.5	0.21	0.13	0.26	0.11	66.3	7.8
76	7	6.1	18.0	0.03	0.06	0.04	0.06	80.1	7.1
94	170	9.4	13.6	0.27	0.04	0.41	0.03	56.7	1.7
106	248	0.0	15.0	0.12	0.01	0.26	0.11	71.0	0.6
107	13	18.7	19.6	0.19	0.08	0.62	0.19	62.7	5.2
116	5	10.0	7.5	0.07	0.94	0.23	0.48	62.3	17.1
123	17	17.7	27.6	0.18	0.13	0.32	0.07	60.9	4.2
128	6	11.4	6.0	0.94	1.38	0.51	0.55	58.3	14.7
132	6	16.1	21.4	0.08	0.05	0.07	0.03	75.9	10.1
135	19	17.1	24.0	0.32	0.11	0.60	0.12	53.4	4.1
139	6	8.5	17.8	0.08	0.08	0.17	0.09	74.0	5.9
144	3	18.5	23.2	3.83	–	1.67	–	30.1	18.7
145	13	10.0	18.8	0.27	0.35	0.22	0.28	52.9	6.5
148	5	10.5	17.8	0.08	0.09	0.07	0.06	76.5	12.3
150	11	12.5	19.7	0.06	0.05	0.07	0.03	78.8	3.5
152	4	12.6	10.6	0.71	–	0.80	–	42.7	29.9
156	4	15.8	22.3	0.10	–	0.20	–	64.2	25.7
157	3	15.6	21.9	0.46	0.50	0.16	0.10	76.5	8.5

of $\beta(\tau)$ (i.e. $\beta \geq 8$), which condition is not satisfied by many groups in Table 2. Therefore, with the model adopted, at least some groups in our sample are not in virial equilibrium.

Furthermore (since $t_f \leq t_0 \leq H_0^{-1}$, which implies $t_f H_0 = \beta t_{cr} H_0 \leq 1$), if all the groups were in the same evolutionary stage and thus had the same $\beta = \beta(\mathrm{fix})$, they should be in early collapse, because $\beta(\mathrm{fix}) \cdot t_{cr}(\mathrm{max}) \cdot H_0 \leq 1$, giving $\beta(\mathrm{fix}) < 1$ (from groups 44, 60, 64, 144 at least).

We have assumed that $z_f > 100$ for our groups and have taken $t_f = t_0$ for all of them. We have chosen the two interesting values $\Omega_0 = 1$ and $\Omega_0 = 0.1$, for which $t_f H_0 = 2/3$ and $t_f H_0 = 0.898$ respectively.

4. Are groups bound systems?

Since some of the chosen groups have virial crossing times (see Table 2) comparable to the Hubble time, one may wonder whether they are bound systems or not.

In principle, one could examine the density excess $\delta\rho/\rho$ of the group with respect to the mean density of the universe. For a bound, spherical system, Gott and Rees (1975) have shown that, at maximum expansion (i.e. for $\tau \approx \pi$),

$$\frac{\delta\rho}{\rho} = \frac{\pi^2}{4\Omega(H_0t_0)^2} - 1\,,$$

which gives values of 30, 22, 4.6 for Ω equal to 0.1, 0.15, and 1.0, respectively. Furthermore, for an unbound system (in an open universe), the maximum value of $\delta\rho/\rho$ turns out to be such that

$$\frac{\delta\rho}{\rho} \le \frac{4}{9\Omega(H_0t_0)^2} - 1$$

which gives values of 23, 4.5, and 2.1 for Ω equal to 0.02, 0.1, and 0.2, respectively.

Groups selected by Geller and Huchra (1983) have a number density exceding 20. So, if number density were proportional to total density, all groups would be bound, unless we live in a very low-density universe. Moreover, most groups would have already gone beyond, or would be very near, the maximum expansion stage. Unfortunately, since we do not know how dark matter is distributed on large scales, the true density excess might be even an order of magnitude lower than the number density, and no compelling conclusion can be drawn from the above argument.

It has been shown by Jackson (1975) that the virial crossing time is not a good indicator to test whether a group is expanding following the Hubble flow. He introduced a crossing time t_J as

$$t_J = R_I/V\,,$$

where V is the usual 3-dimensional velocity dispersion, and R_I is the inertial radius:

$$R_I^2 = \frac{3}{2}\frac{\Sigma_i m_i(x_i^2 + y_i^2)}{\Sigma_i m_i}\,, \tag{12}$$

where m_i are masses, and x_i, y_i the coordinates, projected on the celestial sphere, of galaxies with respect to the center of mass of the system. For a homogeneous sphere, t_J and t_{cr} are equal, but for real systems they generally differ. H_0t_0 is equal to one for a pure Hubble flow. On the hypothesis that groups are still expanding (or in an early stage of evolution), no correlation should exist between masses, velocities and positions. For this reason (see also Mezzetti et al., 1985) velocities have not been weighted by mass in the estimate of V, and the same has been done in the estimate of R_I, setting $m_i = 1$ in Eq. (12). In Table 2 we show H_0t_J for our group sample; groups 64 and 144 have H_0t_J larger than one, while group 152 has $H_0t_J = 0.80$.

Sargent and Turner (1977), in an attempt to estimate Ω, proposed a statistical method useful for checking whether a system is expanding, taking into account all the pairs formed by the members of a group. By using redshifts and positions on the sky, they introduced a parameter, $\langle\theta\rangle$, the mean value of tha angle between a pair separation vector in redshift space and the plane of the sky. They show that, if a system is spherical and follows an unperturbed Hubble flow, $\langle\theta\rangle = 32.^\circ7$. But if the Hubble flow has been slowed by self-gravity (even for an unbound system), then $\langle\theta\rangle < 32.^\circ7$. During collapse (for a bound system) and at virialization $\langle\theta\rangle$ becomes larger than $32.^\circ7$. Equation (4) of Sargent and Turner (1977) allows to estimates $\langle\theta\rangle$ as a function of the ratio H_0/H_p, where H_p is the internal Hubble constant of the system (see the end of this section). Table 2 shows $\langle\theta\rangle$ for our groups; only group 144 has $\langle\theta\rangle < 32.^\circ7$. Notice that this does not necessarily imply that group 144 is unbound, but only that it is not in an advanced stage of collapse.

Another test to check whether a group is bound or not (which is, in a certain sense, a generalization of the two previous ones) is based on the ratio

$$F = \frac{H_0\sigma_d}{\sigma_v} \tag{13}$$

where σ_v and σ_d are the standard deviations of radial velocities (i.e., $V/\sqrt{3}$) and distances from us. For a spherical density enhancement, the Hubble expansion is slowed down by self-gravity by a factor $H_p/H_0 = 1/F$. If a system is unbound, F turns out to be in the range

$$1 \le F \le \frac{3H_0t_0}{2}\,, \tag{14}$$

while, if it is bound, but still expanding,

$$\frac{3H_0t_0}{2} \le F \le \infty\,, \tag{15}$$

H_0t_0 being, as usual, a function of Ω. After maximum expansion, for a homogeneous spherical system, F is equal to $H_0t_{cr} = H_0t_0/\beta$ (see Eq. (11)).

To evaluate σ_d, true galaxy distances (i.e. not deduced from redshifts) are needed. In principle, it would be possible to use Tully-Fisher and Faber-Jackson relations to derive distance modulli, but the data in the literature are still affected by too large errors. For instance, Bottinelli et al. (1984) give a mean error of 0.25 mag, in the B band, for distance modulli derived from H I lines. This corresponds to a $\approx 10\%$ error in distances, that is about 1.5 Mpc for a group at 15 Mpc, larger than the mean size of the system. Since the distribution of distances is the convolution of the true distribution and the error distribution (as it is for radial velocities), too large an uncertainty in distances makes it meaningless, at present, to attempt to estimate σ_d in this way. An error in distance moduli smaller than 0.1 mag is required in order to obtain an error smaller than $\approx 3\%$ in distances (≈ 0.5 Mpc at 15 Mpc).

However, it is possible to overcome this problem if one assumes that groups are spheroidal. In this case σ_d can be estimated from the distribution on the celestial sphere of galaxies in the group; it turns out that $\sigma_d \approx R_I/\sqrt{3}$, so that $F \approx H_0t_J$. As stated above, $\langle\theta\rangle$ is linked with the ratio $H_0/H_p = F$. We have seen that, for a spherical system $F = H_0t_J$, and for a spherical homogeneous system F is also equal to H_0t_{cr}. It is thus possible, in principle, to check the consistency between these three parameters (see Sect. 6).

5. Estimate of uncertainties

In order to evaluate the accuracy related to the parameters introduced in the previous sections (i.e., H_0t_{cr}, H_0t_J, $\langle\theta\rangle$), we have used the Jackknife estimate of standard deviation (see, e.g., Efron, 1982). Each parameter, p, is calculated for all possible subsets of $N-1$ galaxies (N is the total number of seen members in a group). Let p_i be the parameter obtained by deleting the i-th galaxy, and let $p_\bullet$ be the mean value of the N p_i-values. The

Table 3. Values of the kinetic-to-potential energy ratio α, the evolutionary stage τ and the decimal logarithm of the mass-to-light ratio $\log M/L$ for the groups in our sample

	$\Omega = 0.1$	$H_0 t_f = 0.898$		$\Omega = 1.0$	$H_0 t_f = 0.667$	
n_g	α	τ	$\log M/L$	α	τ	$\log M/L$
43	0.25	4.9	3.42	0.20	4.6	3.53
44	0.06	3.8	4.85	0.04	3.6	5.03
46	0.41	5.5	2.06	0.34	5.2	2.14
52	0.60, 0.55, 0.50	6.3, 7.8, 9.6	2.10, 2.14, 2.18	0.54	6.0	2.14
58	0.50	17.3	2.60	0.65, 0.62, 0.50	6.5, 7.4, 12.8	2.48, 2.50, 2.60
60	0.08	4.0	2.37	0.05	3.8	2.53
61	0.28	5.0	2.98	0.21	4.7	3.09
63	0.41	5.5	4.17	0.34	5.2	4.25
64	0.06	3.8	3.64	0.04	3.6	3.83
66	0.15	4.4	3.05	0.11	4.2	3.19
67	0.50	19.1	2.19	0.67, 0.65, 0.50	6.6, 7.3, 14.2	2.06, 2.07, 2.19
68	0.60, 0.54, 0.50	6.3, 7.9, 9.4	2.44, 2.48, 2.51	0.54	6.0	2.48
71	0.44	5.6	2.56	0.37	5.3	2.72
76	0.50	25.7	3.36	0.50	19.1	3.36
94	0.38	5.4	3.25	0.31	5.1	3.34
106	0.56	6.1	2.89	0.50	5.8	2.94
107	0.46	5.7	2.73	0.39	5.4	2.80
116	0.66, 0.64, 0.50	6.6, 7.4, 13.6	1.69, 1.71, 1.81	0.61, 0.56, 0.50	6.3, 7.7, 10.1	1.72, 1.76, 1.81
123	0.47	5.7	2.02	0.41	5.5	2.08
128	0.13	4.3	2.82	0.09	4.1	2.97
132	0.63, 0.59, 0.50	6.4, 7.6, 11.2	3.01, 3.04, 3.11	0.57	6.2	3.05
135	0.34	5.2	2.96	0.28	5.0	3.05
139	0.62, 0.58, 0.50	6.4, 7.7, 10.7	1.89, 1.92, 1.99	0.57	6.1	1.93
144	0.02	3.5	3.19	0.01	3.4	3.42
145	0.38	5.4	3.28	0.31	5.1	3.36
148	0.63, 0.60, 0.50	6.4, 7.6, 11.5	1.72, 1.74, 1.82	0.58, 0.51, 0.49	6.2, 8.2, 8.8	1.76, 1.81, 1.83
150	0.67, 0.65, 0.50	6.6, 7.3, 14.0	2.20, 2.93, 3.04	0.62, 0.57, 0.50	6.4, 7.7, 10.4	2.95, 2.99, 3.04
152	0.18	4.6	2.65	0.13	4.3	2.79
156	0.59, 0.53, 0.50	6.2, 8.0, 9.0	1.98, 2.03, 2.05	0.53	6.0	2.03
157	0.26	4.9	2.73	0.20	4.7	2.84

Jackknife estimate of the standard deviation, σ_{JK}, is given by

$$\sigma_{JK}^2 = \frac{N-1}{N} \Sigma_i [p_i - p_\bullet]^2 . \tag{16}$$

Table 2 contains σ_{JK}-values for $H_0 t_{cr}$, $H_0 t_J$, and $\langle\theta\rangle$. The missing values correspond to groups for which at least one of the $N-1$ subgroups has a correction for the velocity dispersion (due to errors in radial velocity determination) exceeding the dispersion in radial velocities (see Mezzetti et al., 1985).

6. The results

By inspection of Table 2 (taking into account the values of $H_0 t_J$, $\langle\theta\rangle$, and their accuracies) we see that groups 64, 114, and 152 might be in expansion (and, maybe, unbound).

Moreover, the values of $H_0 t_{cr}$, $H_0 t_J$, and $\langle\theta\rangle$, agree (see Sect. 4), when taking into account uncertainties. Only group 44 (one of those without uncertainty on crossing time) shows a questionable behaviour. (However, we notice that groups with 3 or 4 members may depart from spherical symmetry, making less stringent the correspondence between $H_0 t_{cr}$, $H_0 t_J$, and $\langle\theta\rangle$.)

Table 3 lists the group number n_g, the kinetic-to-potential energy ratio α, the evolutionary stage τ and the decimal logarithm of the mass-to-light ratio $\log M/L$ (obtained using Eq. (4) and on the hypothesis that the luminosity evolution of the groups is negligible). At the top of the table, the values of $t_f H_0$ and Ω are given; since $t_f H_0 = H_0 t_0$ for $z_f > 100$, we have assumed that $t_f H_0 = 2/3 = 0.667$ and $t_f H_0 = 0.898$ for the two interesting values $\Omega_0 = 1$ and $\Omega_0 = 0.1$ respectively. For some groups, three figures are reported in Table 3, corresponding to 3-fold solutions to Eq. (11). They represent different phases in the evolution of the system: the first refers to the collapse phase, the second to re-expansion and the third to the relaxed system.

Using the virial mass, the median value of the mass-to-light ratio turns out to be 360 (confidence interval at 90%: 160–540) for the complete sample, and 360 (150–620) for the culled sample obtained by removing groups 44, 64, 114, 152. Taking into account the correction introduced by Eq. (4), for $\Omega = 1$, the median values of M/L are 780 (340–1130) and 660 (300–1130) for the complete and the culled samples, respectively. For $\Omega = 0.1$, the values are 600 (400–1100) and 540 (230–960). The correction due to the evolutionary stage tends to increase the mass-to-light ratio,

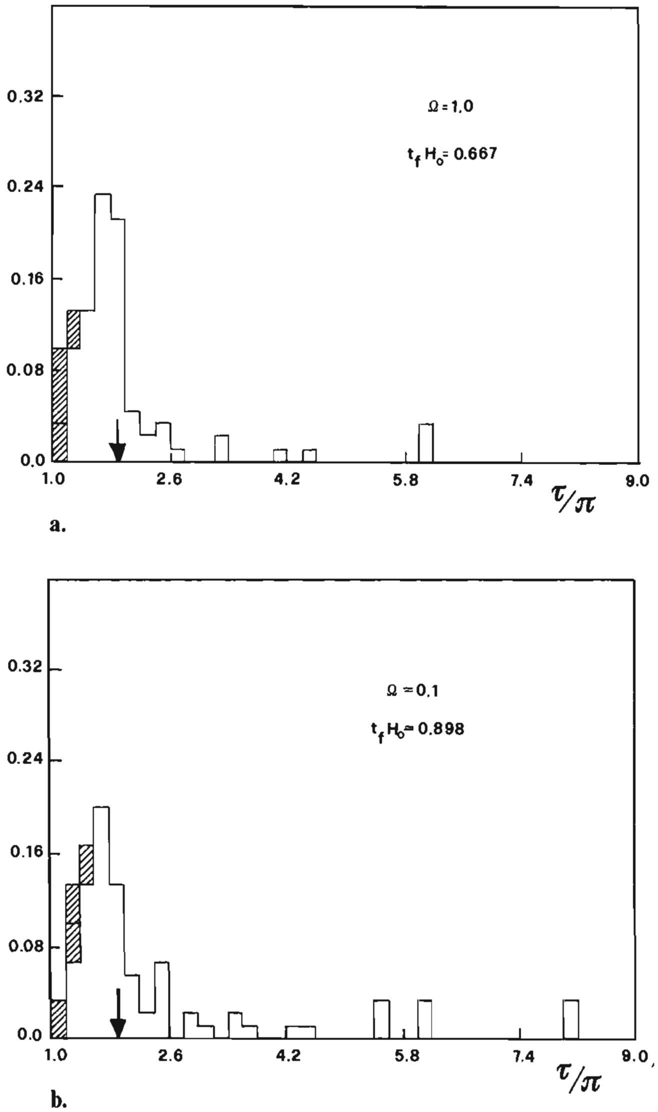

Fig. 3a and b. Histograms of the evolutionary stage τ. The 1-fold solutions are counted three times and all the three values of the 3-fold solutions are considered in solving Eq. (11)

but the differences are not significant, owing to the large spread of M/L values.

Figures 3a and 3b present the histograms of the evolutionary stage τ. Here, the 1-fold solutions are counted three times and all the three values of the 3-fold solutions are considered in order to avoid ambiguity in solving Eq. (11). The dashed area represents the contribution due to groups 44, 64, 144, 152. It is apparent that most ($\approx 75\%$) of the groups have $\tau < 2\pi$, which indicates that they are still in the phase of collapse, and not yet virialized.

To check the sensitivity of the method to the evolutionary model adopted, we have also used the model by Peebles (1970), which describes the behaviour of a system of 300 objects, and one of the simulations by Yabushita and Allen (1985), with 204 objects and non-zero kinetic energy at the moment of maximum expansion ($\alpha_m = 0.25$). The trends for the results obtained with the different models do not differ significantly.

The Virgo cluster lies at the centre of the systems studied and its evolutionary stage (indicated by an arrow in Fig. 3) falls near the peak of the τ distribution in all the models of our choice.

7. Conclusions

We have proposed a method for estimating the evolutionary stage of a self-gravitating system.

According to this method, most of the groups in Geller and Huchra's (1983) catalogue which belong to the Local Supercluster are still collapsing.

Their masses and mass-to-light ratios prove to be slightly greater than those obtained by the straightforward application of the virial theorem.

The evolutionary stage of the Virgo cluster turns out to correspond to the mean evolutionary stage of the groups surrounding it; this suggests that structures different by at least an order of magnitude in mass may be coeval.

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